Logic, relations, number theory, sequences, combinatorics, graphs

Logic

In [mathematics](https://en.wikipedia.org/wiki/Mathematics) and [logic](https://en.wikipedia.org/wiki/Logic), a **direct proof** is a way of showing the [truth](https://en.wikipedia.org/wiki/Truth) or falsehood of a given statement by a straightforward combination of established facts, usually [axioms](https://en.wikipedia.org/wiki/Axiom), existing [lemmas](https://en.wikipedia.org/wiki/Lemma_%28mathematics%29) and [theorems](https://en.wikipedia.org/wiki/Theorem), without making any further assumptions.

en.wikipedia.org/wiki/Direct\_proof

**Proof by exhaustion**, also known as **proof by cases**, **proof by case analysis**, **complete induction** or the **brute force method**, is a method of [mathematical proof](https://en.wikipedia.org/wiki/Mathematical_proof) in which the statement to be proved is split into a finite number of cases or sets of equivalent cases, and where each type of case is checked to see if the proposition in question holds. This is a method of [direct proof](https://en.wikipedia.org/wiki/Direct_proof).

en.wikipedia.org/wiki/Proof\_by\_exhaustion

Indirect proof

In [logic](https://en.wikipedia.org/wiki/Logic), **proof by contradiction** is a form of [proof](https://en.wikipedia.org/wiki/Mathematical_proof) that establishes the [truth](https://en.wikipedia.org/wiki/Truth#Formal_theories) or the [validity](https://en.wikipedia.org/wiki/Validity_%28logic%29) of a [proposition](https://en.wikipedia.org/wiki/Proposition), by showing that assuming the proposition to be false leads to a [contradiction](https://en.wikipedia.org/wiki/Contradiction). Although it is quite freely used in mathematical proofs, not every [school of mathematical thought](https://en.wikipedia.org/wiki/Philosophy_of_mathematics) accepts this kind of [nonconstructive proof](https://en.wikipedia.org/wiki/Nonconstructive_proof) as universally valid.

More broadly, proof by contradiction is any form of argument that establishes a statement by arriving at a contradiction, even when the initial assumption is not the negation of the statement to be proved. In this general sense, proof by contradiction is also known as **indirect proof**

en.wikipedia.org/wiki/Proof\_by\_contradiction

In [logic](https://en.wikipedia.org/wiki/Logic), the [**contrapositive**](https://en.wikipedia.org/wiki/Contraposition) of a [conditional](https://en.wikipedia.org/wiki/Indicative_conditional) statement is formed by negating both terms and reversing the direction of inference. More specifically, the contrapositive of the statement "if *A*, then *B*" is "if not *B*, then not *A*." A statement and its contrapositive are logically equivalent, in the sense that if the statement is true, then its contrapositive is true and vice versa.

en.wikipedia.org/wiki/Proof\_by\_contrapositive

Relations

A recurrence relation is an equation that defines a sequence based on a rule that gives the next term as a function of the previous term(s).

f(f(x))

**Iteration** is the repetition of a process in order to generate a (possibly unbounded) sequence of outcomes. Each repetition of the process is a single iteration, and the outcome of each iteration is then the starting point of the next iteration.

Number theory

The quotient-remainder theorem says that when any integer n is divided by any positive integer d, the result is a quotient q and a nonnegative remainder r that is smaller than d.

n = dq + r and 0 ≤ r < d.

5 = 2×2+1

The fundamental theorem of arithmetic states that every positive integer (except the number 1) can be represented in exactly one way as a product of one or more primes. This theorem is also called the unique factorization theorem.

Public key cryptography

Question:

Use the rule: $If b^{p} mod n=r and b^{q} mod n=m, then m^{p} mod n= r^{q} mod n$

to publicly pass secret information.

Use p and q as your private keys, these are secret numbers for you and your partner.

b, n, r, m are public numbers, everybody can know these numbers.

planetcalc.com/8326/

Sequences

n2 << 2n << n! << nn, n $\rightarrow \infty $

Question:

Find the equation for the sequence: 1, 4, 9, 16, 25, 36, 49, …

 3, 5, 7, 9, 11, 13

 2, 2, 2, 2, 2

x(n) =n^2 + bn + c

x(1) = 1 = 1+b+c

x(2) = 4 = 4 +2b+c

b=c=0

x(n)=n^2

Fibonacci numbers

112358….

$F\_{n}=\frac{φ^{n}-p^{n}}{φ-p}=\frac{φ^{n}-p^{n}}{\sqrt{5}}$ (11)

$φ=\frac{1+\sqrt{5}}{2}$ (12)

$p=\frac{1-\sqrt{5}}{2}$ (13)

youtube.com/watch?v=ITSbuT9ojOw

$F\_{n}=F\_{n-1}+F\_{n-2}$ (1)

Substituting $F\_{n}=x^{n}$ (2)

Plugging (2) to (1), we get:

$x^{n}=x^{n-1}+x^{n-2}$ (3)

Dividing (3) by $x^{n-2}$, we get

Characteristic equation:

$x^{2}-x-1=0$ (14)

$$x\_{1}=\frac{1+\sqrt{5}}{2}$$

$$x\_{2}=\frac{1-\sqrt{5}}{2}$$

Question:

Calculate Fibonacci number L.

L = s mod 10

s is your student number.

Dim F(9)

s = 99000004

L = s Mod 10

fi = (1 + Sqr(5)) / 2

p = (1 - Sqr(5)) / 2

F(L) = (fi \* fi \* fi \* fi - p \* p \* p \* p) / Sqr(5)

F(L) = (fi^L - p^L) / Sqr(5)

MsgBox F(L)

In computer science, a Fibonacci heap is a data structure for priority queue operations, consisting of a collection of heap-ordered trees. It has a better amortized running time than many other priority queue data structures including the binary heap and binomial heap.

A **binary heap** is a [heap](https://en.wikipedia.org/wiki/Heap_%28data_structure%29) [data structure](https://en.wikipedia.org/wiki/Data_structure) that takes the form of a [binary tree](https://en.wikipedia.org/wiki/Binary_tree). Binary heaps are a common way of implementing [priority queues](https://en.wikipedia.org/wiki/Priority_queue). The binary heap was introduced by [J. W. J. Williams](https://en.wikipedia.org/wiki/J._W._J._Williams) in 1964, as a data structure for [heapsort](https://en.wikipedia.org/wiki/Heapsort).

en.wikipedia.org/wiki/Binary\_heap

In [computer science](https://en.wikipedia.org/wiki/Computer_science), a **binomial heap** is a [data structure](https://en.wikipedia.org/wiki/Data_structure) that acts as a [priority queue](https://en.wikipedia.org/wiki/Priority_queue) but also allows pairs of heaps to be merged. It is important as an implementation of the [mergeable heap](https://en.wikipedia.org/wiki/Mergeable_heap) [abstract data type](https://en.wikipedia.org/wiki/Abstract_data_type) (also called [meldable heap](https://en.wikipedia.org/wiki/Meldable_heap%22%20%5Co%20%22Meldable%20heap)), which is a [priority queue](https://en.wikipedia.org/wiki/Priority_queue) supporting merge operation. It is implemented as a [heap](https://en.wikipedia.org/wiki/Heap_%28data_structure%29) similar to a [binary heap](https://en.wikipedia.org/wiki/Binary_heap) but using a special tree structure that is different from the [complete binary trees](https://en.wikipedia.org/wiki/Complete_binary_tree) used by binary heaps. Binomial heaps were invented in 1978 by [Jean Vuillemin](https://en.wikipedia.org/wiki/Jean_Vuillemin).

en.wikipedia.org/wiki/Binomial\_heap

Question:

Try to do Zimmermann math competition.

azspcs.com

Combinatorics

Question:

In how many ways you can write the digits of your k?

k = s mod 10000

P(n,n)=n!

P(m,n)=m!/(m-n)!

k = 7015, n = 4, 4!

Multiplication rule is used when variables are independent.

Multiplication rule is similar to AND in logic.

If I have 3-digit password, then there will be 1000 options.

We use addition rule when there is OR.

If my password can be from 0 to 3 digits, then I add the number of options:

1 + 10 + 100 + 1000 = $\frac{1-10000}{1-10}=\frac{9999}{9}$=1111

Here we use combination of addition rule and multiplication rule.

Question:

10 fair coins were tossed. How many options? (use multiplication rule)

How many options contain 5 heads? C(10, 5).

How many options contain at least 5 heads?

Use addition principle: C(10, 5)+ C(10, 6)+ C(10, 7)+ C(10, 8)+ C(10, 9)+ C(10, 10)

Question:

Hack password.

https://calculus1only.weebly.com/uploads/5/9/8/5/59854633/password-hacking-game-rules.docx

https://calculus12s.weebly.com/uploads/2/5/3/9/25393482/code4password\_cracki4game.txt

https://calculus1only.weebly.com/uploads/5/9/8/5/59854633/guessinput.txt

P(6,5)=6!=720

You must guess the password made of 5 different digits, selected from 6 different digits.

You cannot repeat digits in the password; every digit in the password is only once.

After every your guess, the computer tells you how many digits are on correct (c) and at wrong (w) places in the password.

The password is guessed if c = 5 and w = 0.

Graphs

Question:

Solve the Graceful Graph Problem for *(e+3)* vertices.

http://discrete4math.weebly.com/uploads/2/5/3/9/25393482/code5better.txt

http://discrete4math.weebly.com/uploads/2/5/3/9/25393482/code6.txt

http://discrete4math.weebly.com/uploads/2/5/3/9/25393482/7code7.txt

http://discrete4math.weebly.com/uploads/2/5/3/9/25393482/8code.txt

http://discrete4math.weebly.com/uploads/2/5/3/9/25393482/9code.txt

http://discrete4math.weebly.com/uploads/2/5/3/9/25393482/10code10.txt

http://discrete4math.weebly.com/uploads/2/5/3/9/25393482/11code11.txt

http://discrete4math.weebly.com/uploads/2/5/3/9/25393482/12code12.txt

http://azspcs.com/Contest/GracefulGraphs